

CB 519  
Construction Project  
Management 2

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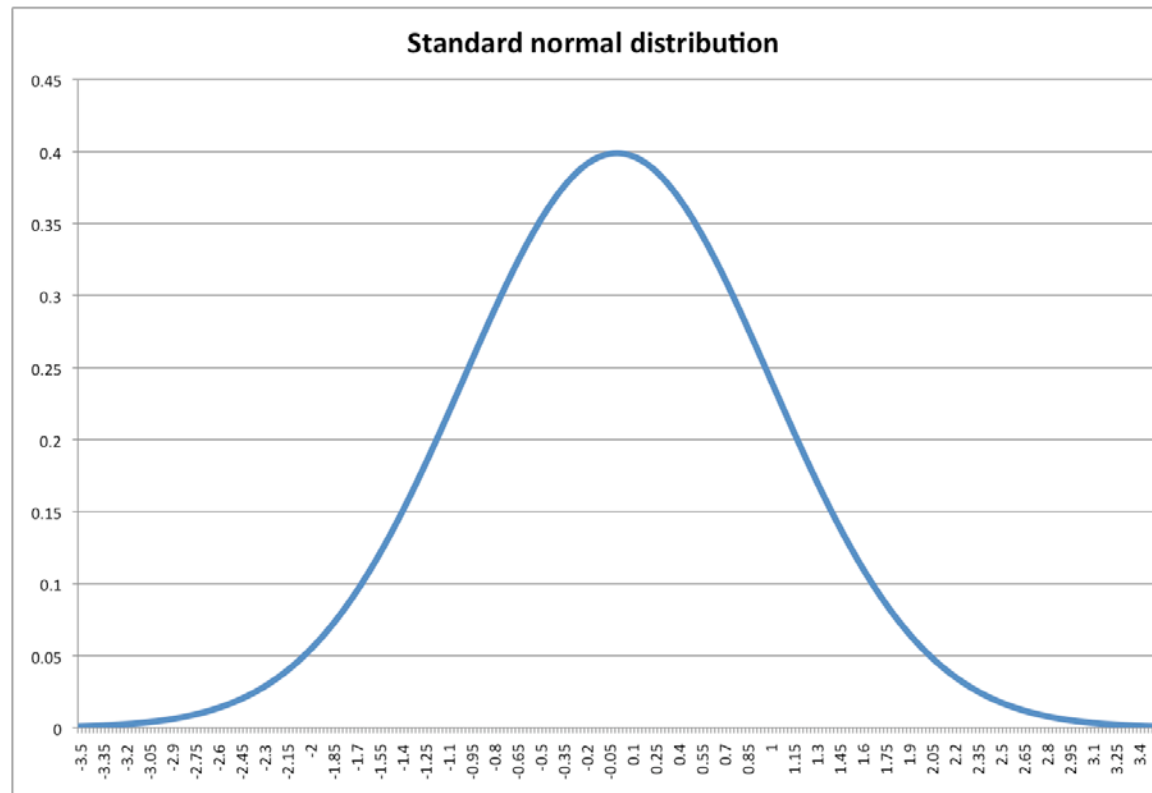
Fall - 2017

# Stochastic Scheduling – Uncertainty

- Activities duration
  - How do we calculate duration?
  - Are production rates deterministic?
  - Assume we have an excavation activity with total duration of 30days. What does the 30days actually means?

# Stochastic Scheduling – Uncertainty

- Stochastic/uncertain activity duration



Can CPM handle such  
uncertainty?

# CPM duration drawback

- CPM is a single and deterministic duration estimate model.
- Such estimate ignores the probabilistic and variability associated with construction.
- Variation can be due to crew's efficiency, weather, management conditions, etc.

# PERT

- Program Evaluation Review Technique (PERT)
- Duration ( $T_e$ ) is calculated through three time estimates
  - Optimistic ( $T_o$ )
  - Most-likely ( $T_m$ )
  - Pessimistic ( $T_p$ )

$$T_e = \frac{T_o + 4T_m + T_p}{6}$$

# Example

| Activity | Predecessor | $T_o$ | $T_m$ | $T_p$ | $T_e$ |
|----------|-------------|-------|-------|-------|-------|
| A        | --          | 1     | 1     | 1     |       |
| B        | A           | 3     | 7     | 11    |       |
| C        | A           | 2     | 6     | 7     |       |
| D        | A           | 1     | 3     | 8     |       |
| E        | B           | 1     | 3     | 5     |       |
| F        | B,C         | 5     | 7     | 9     |       |
| G        | D           | 5     | 8     | 9     |       |
| H        | E,F         | 3     | 7     | 9     |       |
| J        | F           | 2     | 5     | 7     |       |
| K        | F,G         | 3     | 3     | 3     |       |
| L        | H,J,K       | 2     | 5     | 8     |       |

# Example

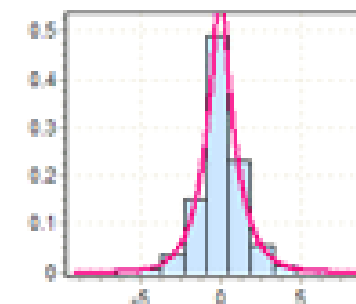
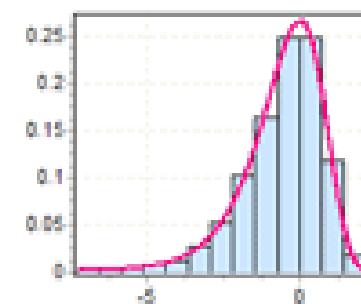
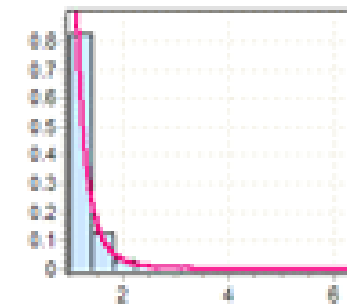
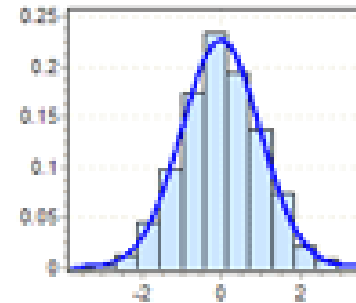
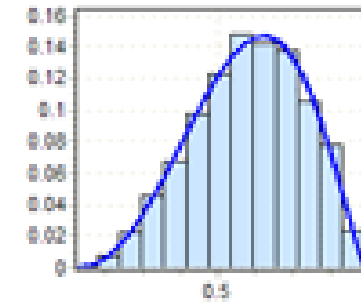
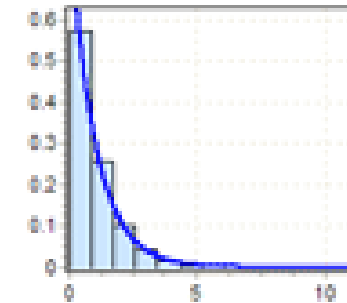
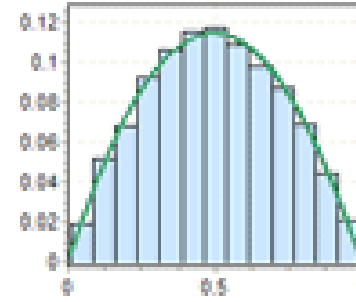
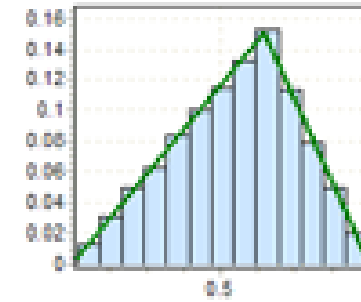
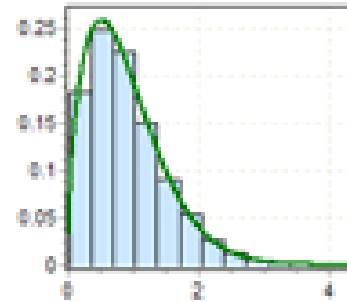
| Activity | Predecessor | $T_o$ | $T_m$ | $T_p$ | $T_e$    |
|----------|-------------|-------|-------|-------|----------|
| A        | --          | 1     | 1     | 1     | 1        |
| B        | A           | 3     | 7     | 11    | 7        |
| C        | A           | 2     | 6     | 7     | 5.5      |
| D        | A           | 1     | 3     | 8     | 3.5      |
| E        | B           | 1     | 3     | 5     | 3        |
| F        | B,C         | 5     | 7     | 9     | 7        |
| G        | D           | 5     | 8     | 9     | 7.666667 |
| H        | E,F         | 3     | 7     | 9     | 6.666667 |
| J        | F           | 2     | 5     | 7     | 4.833333 |
| K        | F,G         | 3     | 3     | 3     | 3        |
| L        | H,J,K       | 2     | 5     | 8     | 5        |

*Still 50% probability*



# Probability

- What is probability?
  - Likelihood that an event will occur
- What is Probability distribution?
  - Probability function of a variable that governs its probability.

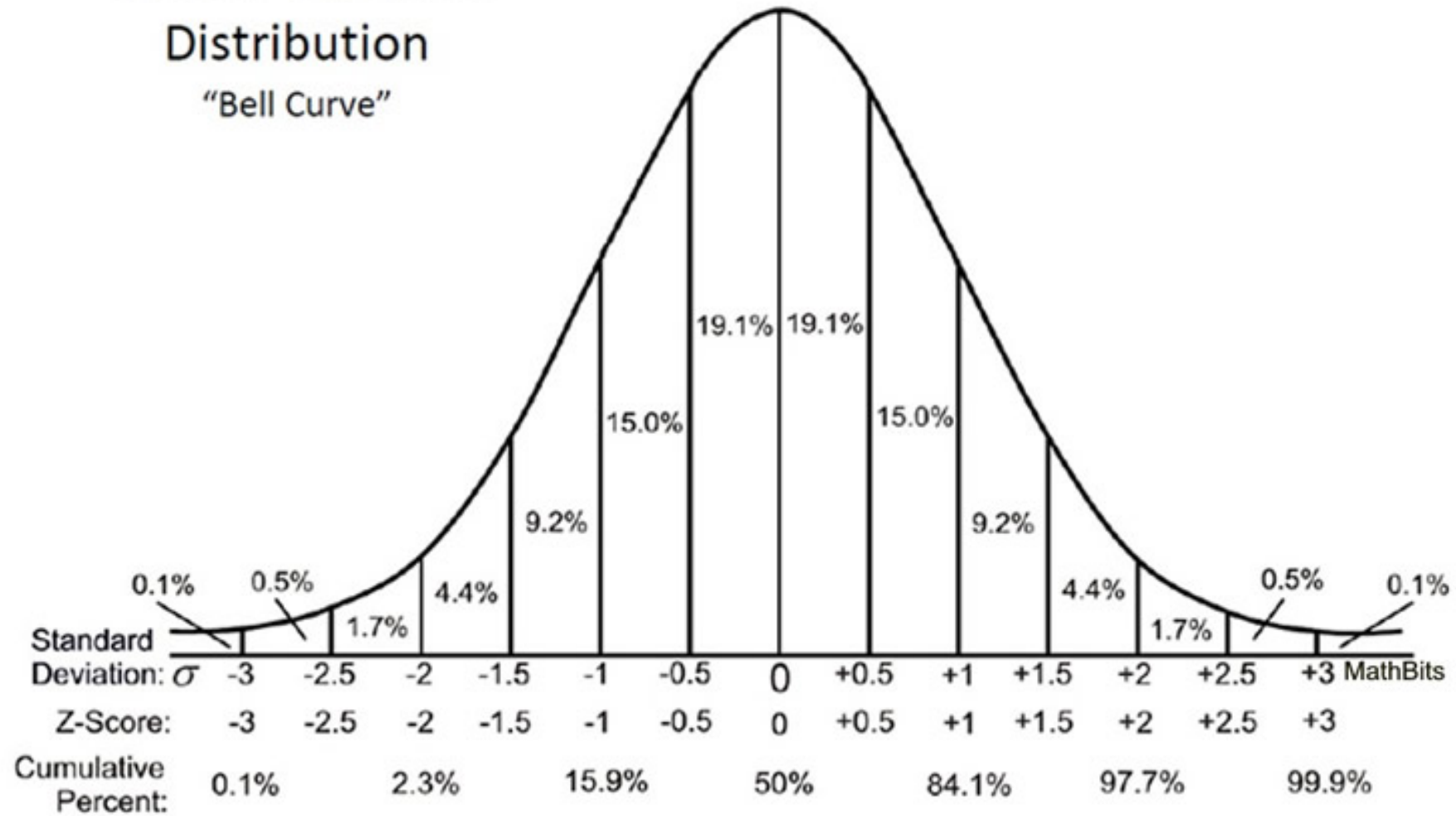


# Exploring probability distributions

- Coin and dice games

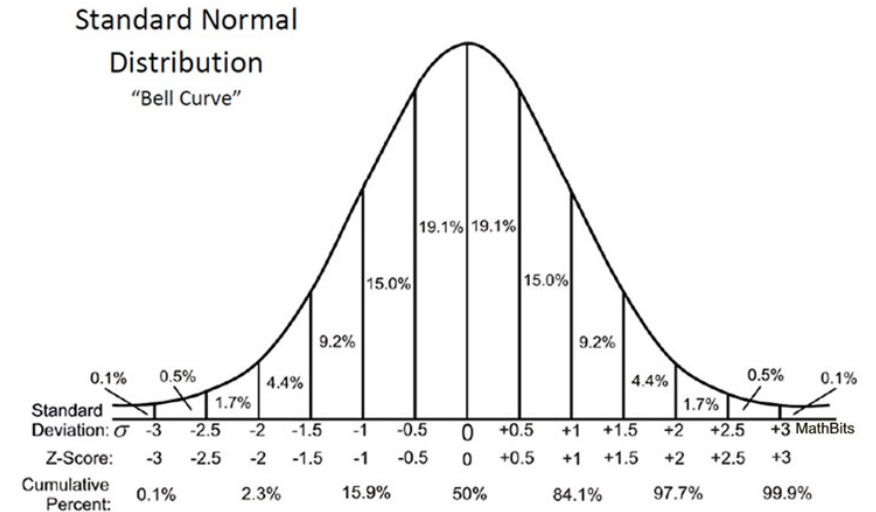
# Probability

Standard Normal  
Distribution  
"Bell Curve"



# Probability

- Mean
  - The average value at 50% probability
- Standard deviation
  - A number that express how much the values of each group differ from the mean
- Variance
  - Describes how are the numbers spread out from the mean.



# Probability

- Mean ( $T_e$ )

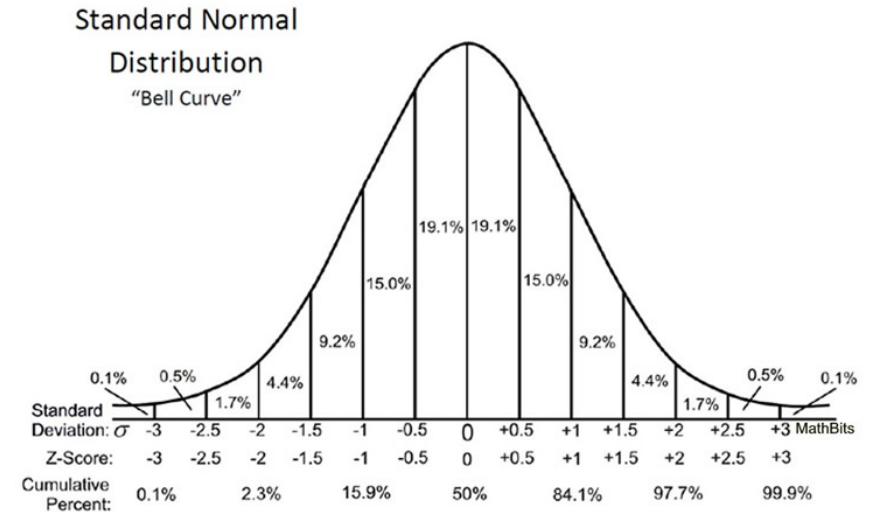
$$T_e = \frac{T_o + 4T_m + T_p}{6}$$

- Standard deviation ( $s$ )

$$s = \frac{T_p - T_o}{6}$$

- Variance ( $v$ )

$$v = s^2$$



# Example

| Activity | Predecessor | $T_o$ | $T_m$ | $T_p$ | $T_e$    | s | v |
|----------|-------------|-------|-------|-------|----------|---|---|
| A        | --          | 1     | 1     | 1     | 1        |   |   |
| B        | A           | 3     | 7     | 11    | 7        |   |   |
| C        | A           | 2     | 6     | 7     | 5.5      |   |   |
| D        | A           | 1     | 3     | 8     | 3.5      |   |   |
| E        | B           | 1     | 3     | 5     | 3        |   |   |
| F        | B,C         | 5     | 7     | 9     | 7        |   |   |
| G        | D           | 5     | 8     | 9     | 7.666667 |   |   |
| H        | E,F         | 3     | 7     | 9     | 6.666667 |   |   |
| J        | F           | 2     | 5     | 7     | 4.833333 |   |   |
| K        | F,G         | 3     | 3     | 3     | 3        |   |   |
| L        | H,J,K       | 2     | 5     | 8     | 5        |   |   |

# Example

| Activity | Predecessor | $T_o$ | $T_m$ | $T_p$ | $T_e$    | s        | v        |
|----------|-------------|-------|-------|-------|----------|----------|----------|
| A        | --          | 1     | 1     | 1     | 1        | 0        | 0        |
| B        | A           | 3     | 7     | 11    | 7        | 1.333333 | 1.777778 |
| C        | A           | 2     | 6     | 7     | 5.5      | 0.833333 | 0.694444 |
| D        | A           | 1     | 3     | 8     | 3.5      | 1.166667 | 1.361111 |
| E        | B           | 1     | 3     | 5     | 3        | 0.666667 | 0.444444 |
| F        | B,C         | 5     | 7     | 9     | 7        | 0.666667 | 0.444444 |
| G        | D           | 5     | 8     | 9     | 7.666667 | 0.666667 | 0.444444 |
| H        | E,F         | 3     | 7     | 9     | 6.666667 | 1        | 1        |
| J        | F           | 2     | 5     | 7     | 4.833333 | 0.833333 | 0.694444 |
| K        | F,G         | 3     | 3     | 3     | 3        | 0        | 0        |
| L        | H,J,K       | 2     | 5     | 8     | 5        | 1        | 1        |

# Stochastic properties of critical path

- Duration of critical path

$$T_{project} = \sum T_{e_{CP}}$$

- Variance of critical path

$$V_{project} = \sum V_{CP}$$

- Standard deviation of critical path

$$S_{project} = \sqrt{V_{project}}$$



# What did we gain from this?

- Even though we still have  $T_e$  at 50% probability, we have better understanding on the likelihood of this estimation.
- Through the standard deviation and variance, we can predict the probability of finishing the activities on time, given the changes in any activity.

# Another advantage of probabilistic distribution

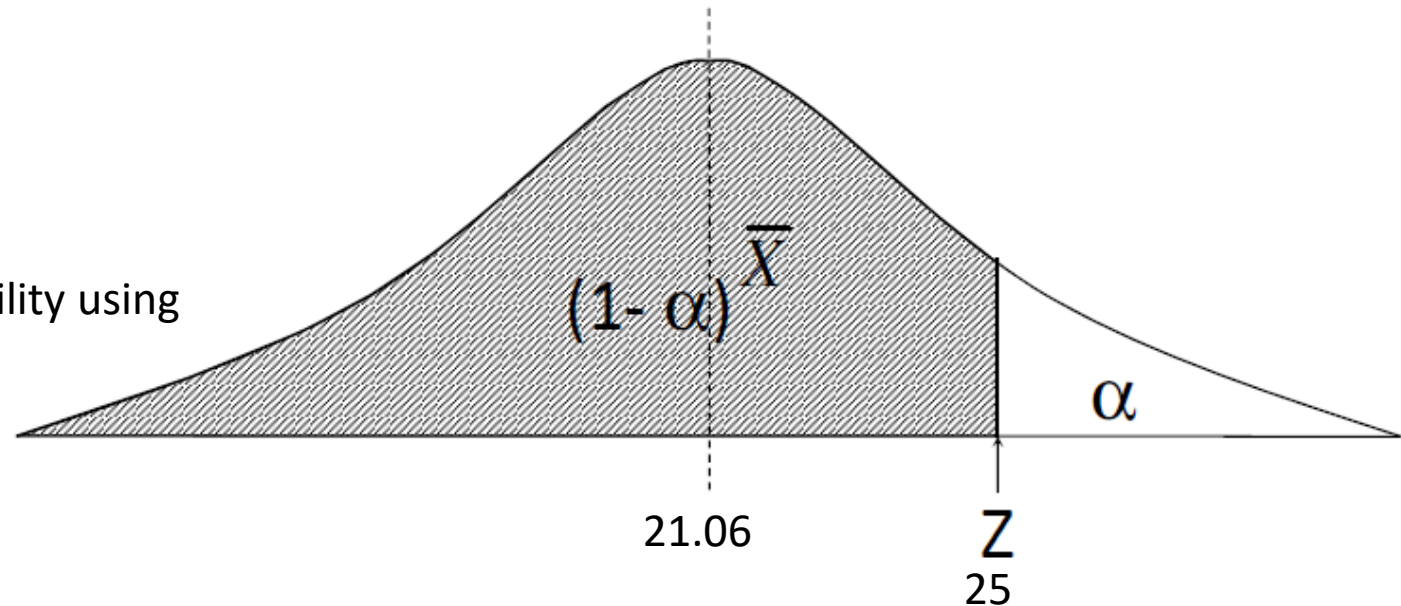
- Since we have the properties of the stochastic project, we can evaluate the probability of completing the project (or a task) at a given date.

# Probability of completing before a given time.

What if we want to check the probability of finishing the project before 25 days?

$$Z = (T_{\text{new}} - T_e) / s$$

Then we can calculate the probability using the following table



| <i>z</i> | 0.00  | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0      | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1      | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2      | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3      | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4      | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5      | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6      | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7      | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8      | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9      | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0      | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1      | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2      | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3      | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4      | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5      | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6      | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7      | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8      | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9      | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0      | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1      | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2      | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3      | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4      | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5      | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |

# More examples

- What is the probability of finishing before 23 days?
- What is the probability of finishing before 19 days?
- What is estimated project duration if we want to finish with probability 75%?