

# CB 714

# Advanced System Analysis

Dr. Mohamed Saeid Eid

Spring - 2018

# Resume

## Dr. Mohamed Saeid Eid

- BS and MS in construction Engineering, AAST (2008, and 2012, respectively)
- PhD in Civil and Environmental Engineering, Construction focus, University of Tennessee, Knoxville (2017)

# Research Interest

- Multi-objective optimization
  - Repetitive Activities Scheduling
  - Site layout
- Simulation
  - Disaster Recovery and Community Vulnerability
  - Traffic Behavior
  - Construction performance
- Game Theory
  - Traffic behavior
  - Construction bidding
  - Collaborative construction projects

# Syllabus

- What do you expect to learn?
  - Advanced LP modeling
  - Basic java programming skills
  - Game theory modeling and intuition
- What do I expect from my students?
  - Attention and participation
  - Curiosity to learn
  - Research and readings

# Syllabus - Content

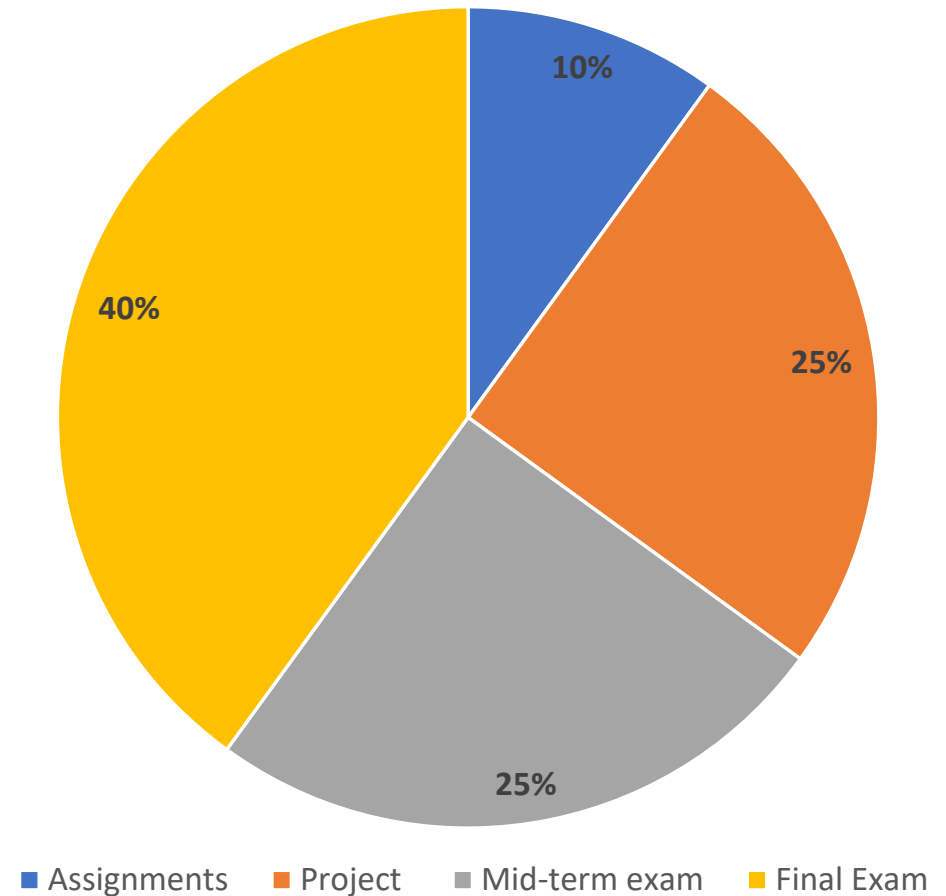
Week#	Title	readings
1	Introduction, mathematical linear programming models	
2	Solving LP	Hillier (pages 7-17)
3	Multi criteria decision analysis	Anderson et al. (Pages 594-624)
4	Intro to programming for research	Richard Feynman Computer Heuristic Lecture (YouTube)
5	Loops and condition functions	
6	Classes and objects	
7	7 <sup>th</sup> week exam	
8	Intro to game theory	
9	Static single shot games strategies	Gibson
10	Static single shot games strategies	
11	Mixed strategies	Turocy and von Stengel (pages 1-22)
12	12 <sup>th</sup> week exam	
13	Evolutionary game theory	
14	Project Presentation	
15	Project presentation	

# Recommended text books and references

- Hillier, F., (2001). Introduction to Operation Research, 7<sup>th</sup> edition.
- Anderson et al. (2012). An introduction to Management Science, 2<sup>nd</sup> edition.
- Schildt (2014). Java, A beginner's guide, 6<sup>th</sup> edition.
- Gibson (1992). Game theory for applied economics.

# Syllabus – Grading

- 30% on the 7<sup>th</sup> week
  - 5% participation
  - 10% assignments/projects
  - 15% exam
- 20% on the 12<sup>th</sup> week
  - 10% assignments
  - 10% exam
- 10% project presentation
- 40% final exam



# Term Project

- A group of max 3 students
- Topic related to your research
- Using one of the main topics taught in class
- Paper of minimum 5 pages
- A 10 minute presentation

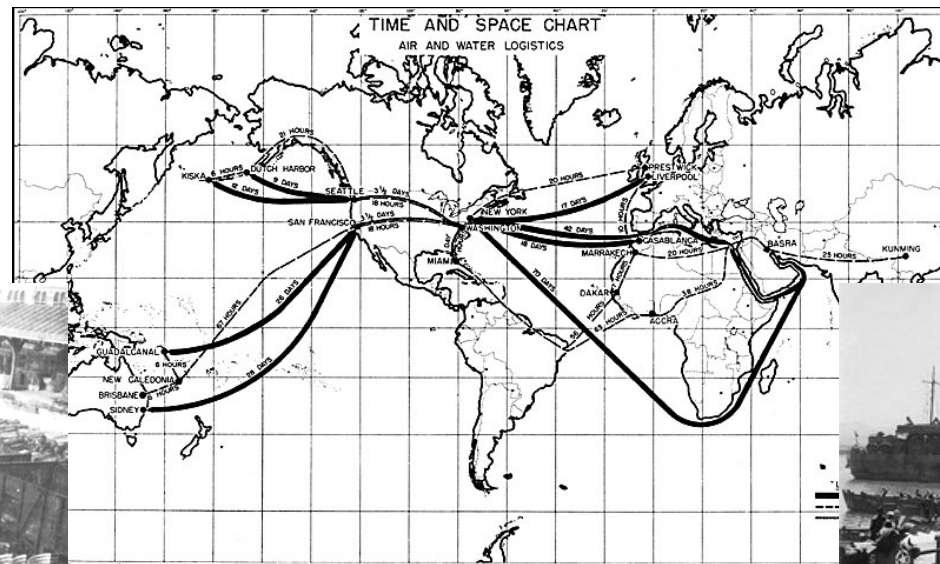


# Class rules

- Contact
  - No phone calls. Each call worth -5% of your grade.
  - meid@aast.edu
  - Use an appropriate subject title, and English language only
  - Website: Msaeideid.com
- Late assignments
  - No late assignments are accepted beyond due date
- Teamwork
- Class ethics and *Academic Honesty*
- NO CELL PHONES!

# Introduction

- History of System Analysis and Operation Research



# Introduction

- CPM
- PERT
- LOB
- Network theory
- Game theory

# Mathematical Models

- Why?

$$F=ma$$

- Modeling Approach:
  - Define the problem of interest and collect the relevant data
  - Formulate the problem mathematically to represent its main parameters

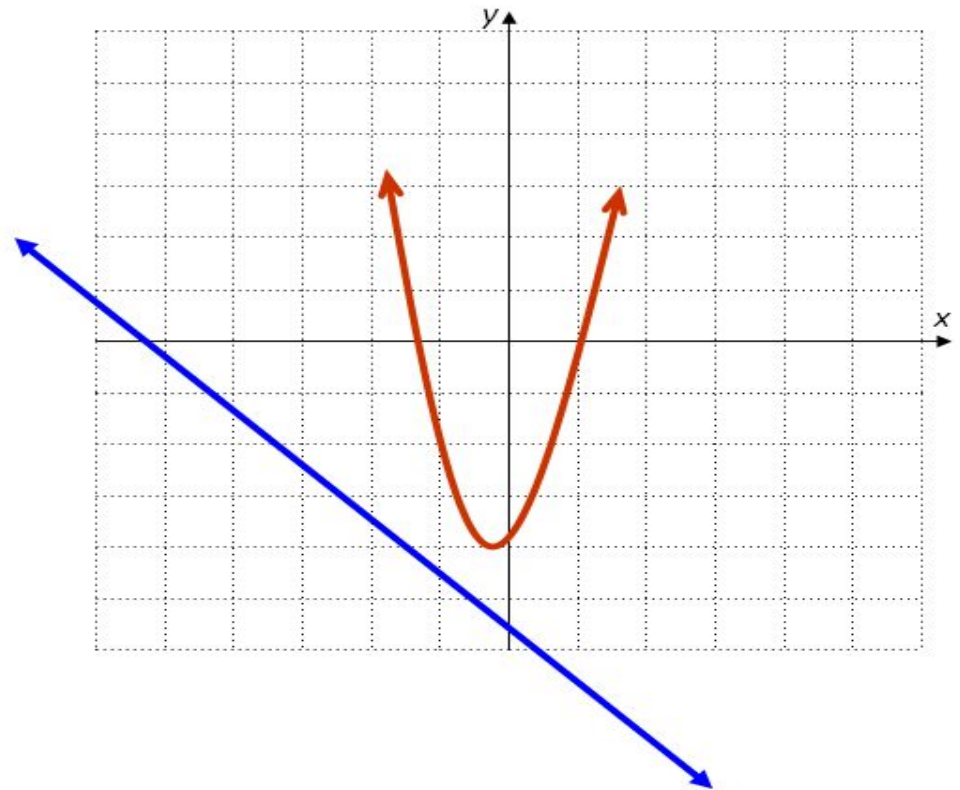
# Mathematical Models

- Variables
- Objectives
- Constraints

# Linear Programming

Not Linear

Linear



# Linear Programming

- Inequalities and linearity

$$\text{Max } 3X_1 + 5X_2 - 3X_3$$

$$\text{S.T. } 2X_1 + X_2 \leq 12$$

$$X_2 - X_3 > 10$$

$$\text{Max } 3X_1 + 5X_2 - 3X_3$$

$$\text{S.T. } 2X_1 + X_2 \leq 12$$

$$X_2 - X_1 X_3 > 10$$

# Limitation of LP

- Linear programming does not account for uncertainty
- Assumption of constant return
- Assumption of linear relationship between input and out



# Examples

- As a site engineer, you want to procure the needed aggregates from different gravel and sand pits around your site with minimum cost. If each cubic meter of aggregates from the Pit A costs LE 300, while for Pit B, costs LE 400.
- Pit A aggregates has a mix of  $\frac{1}{2}$  sand,  $\frac{1}{2}$  gravel.
- Pit B aggregates is 100% sand
- The design requires a minimum of 30% sand.
- You are required to procure a minimum of 1000 cubic meter of aggregate.
- Formulate this problem using LP.

# Examples

- A project manager is concerned about assigning excavation crews to different job locations. If currently the manager has three excavation crews of different production rates and three job sites of different conditions. The following table shows the time (day) each crew will take to finish working per job site.

	Job 1	Job 2	Job 3
Ex. 1	5	3	4
Ex. 2	6	4	7
Ex. 3	5	3	6

- Formulate this problem using LP

# Suggested readings

- Srour, I.M., Haas, C.T., and Morton, D.P. (2006). Linear Programming Approach to Optimize Strategic Investment in the Construction Workforce. *Journal of Construction Engineering and Management* 132, 1158–1166.
- de Athayde Prata Bruno, Pitombeira-Neto Anselmo Ramalho, and de Moraes Sales Caio Jucá (2015). An Integer Linear Programming Model for the Multiperiod Production Planning of Precast Concrete Beams. *Journal of Construction Engineering and Management* 141, 04015029.
- Vimonsatit V., Tan K. H., and Ting S. K. (2003). Plastic Limit Temperatures of Flexibly Connected Steel Frames: A Linear Programming Problem. *Journal of Structural Engineering* 129, 79–86.
- Price Eyal, and Ostfeld Avi (2014). Discrete Pump Scheduling and Leakage Control Using Linear Programming for Optimal Operation of Water Distribution Systems. *Journal of Hydraulic Engineering* 140, 04014017.

# Other Mathematical Notations

$$\forall x \in X, x > 0$$

- All  $x$  in the set  $X$  are non-negative

$$\exists x \in X \text{ such that } x < 0$$

- There is an  $x$  in the set of  $X$  such that its value is negative

$$\exists x \in X \text{ such that } \neg Q(x)$$

- There is an  $x$  in the set of  $X$  such that the function  $Q(x)$  is not true

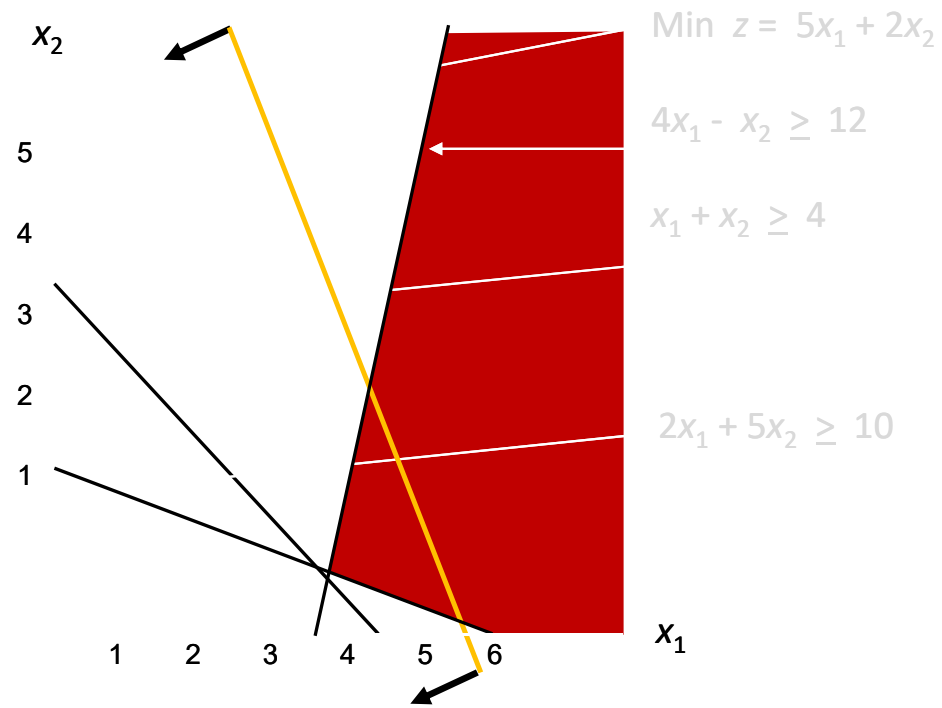
# Solving LP Problems

- Given the developed LP model, we can determine the feasible region of the solution space.
- A feasible region is the boundaries for the variables values to determine an output for the problem that respects all the constraints.
- An optimal solution(s) can be then determined within such feasible region.

# Graphical Method

- The graphical method is ideal for two variables
- Each axis represents a variable
- The constraints are drawn to determine the feasible region
- The optima solution can then be tested in multiple approaches
  - Extreme points
  - Objective function slope

# Illustration

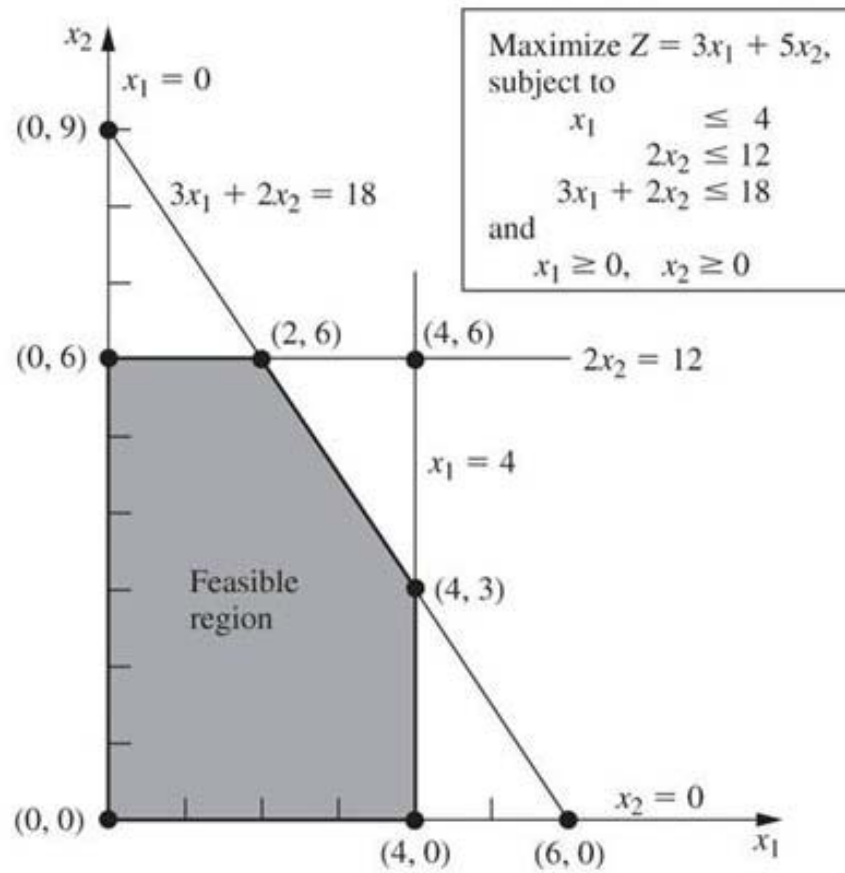


# Example

- As a construction developer, you are trying to maximize your profit from constructing two types of compounds; multi-story buildings, and single homes. Each of the multi-story compounds generates EGP3 Million profit, while the single homes generates EGP5 million profit.
- The acquired lands you possess can only construct up to four multi-story compounds.
- Meanwhile, the planning and designing of the single home compounds require 2 month each, and you only have up to 12 month of design.
- To construct, you need 3 and 2 site engineers for the multi-story building and single homes compounds, respectively. You only have 18 site engineers available.

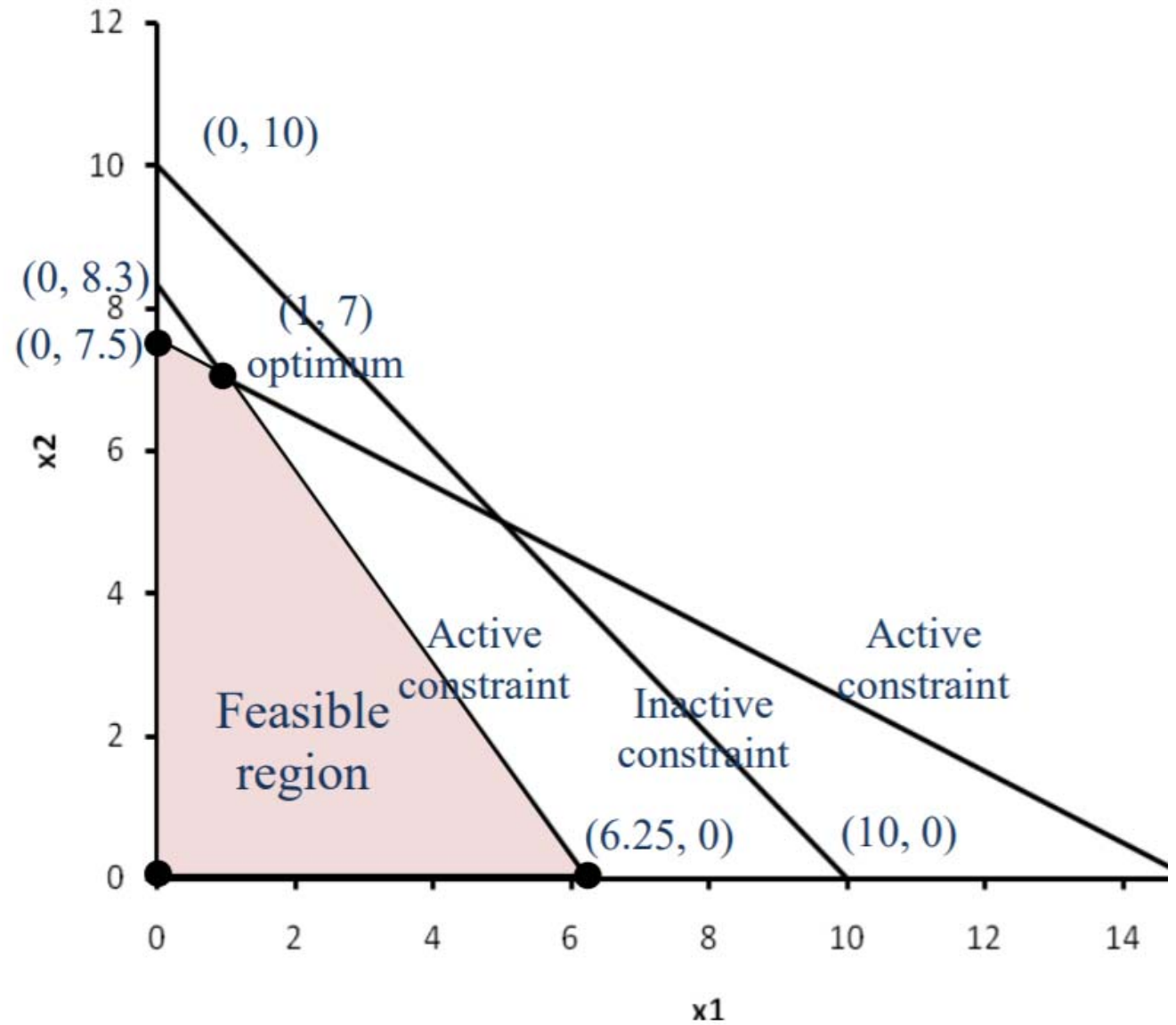


# Example

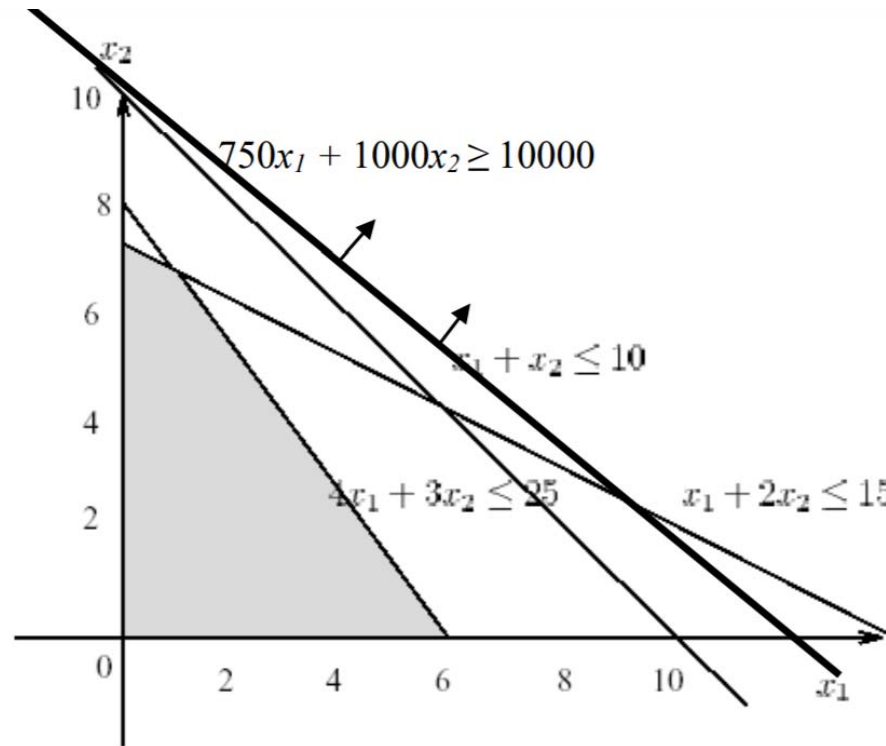


# Constraints

- Active constraints
- Inactive constraints



# Infeasible problem



# Simplex Method

- We can generalize any linear programming problem to the following format to deal with any number of variables:

$$\begin{array}{ll} \textit{Max} & c_1x_1 + c_2x_2 + \dots + c_nx_n \\ \textit{subject to} & a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ & \dots\dots\dots \\ & a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \\ & x_1 \geq 0; \dots\dots x_n \geq 0 \end{array}$$

# Simplex Method

- Slack variables and solution search

$$\text{Min} \quad -2x_1 + 3x_2$$

$$x_1 - 3x_2 + 2x_3 \leq 3$$

$$-x_1 + 2x_2 \geq 2$$

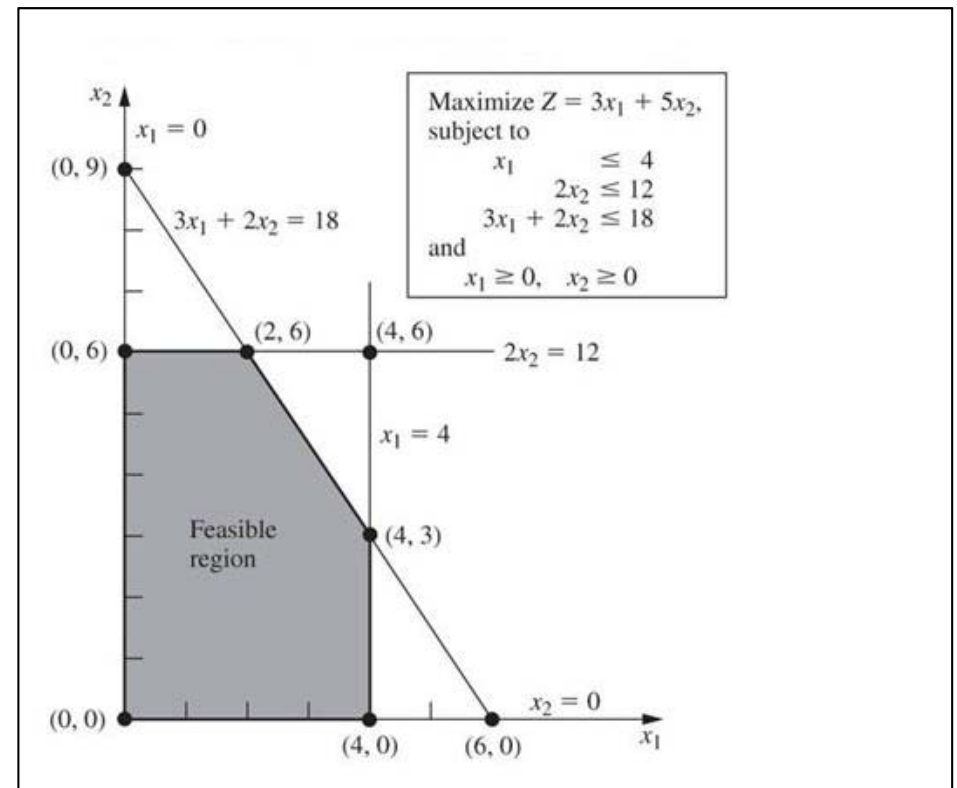
$$\text{Max} \quad 2x_1 - 3x_2$$

$$x_1 - 3x_2 + 2x_3 + s_1 = 3$$

$$-x_1 + 2x_2 - s_2 = 2$$

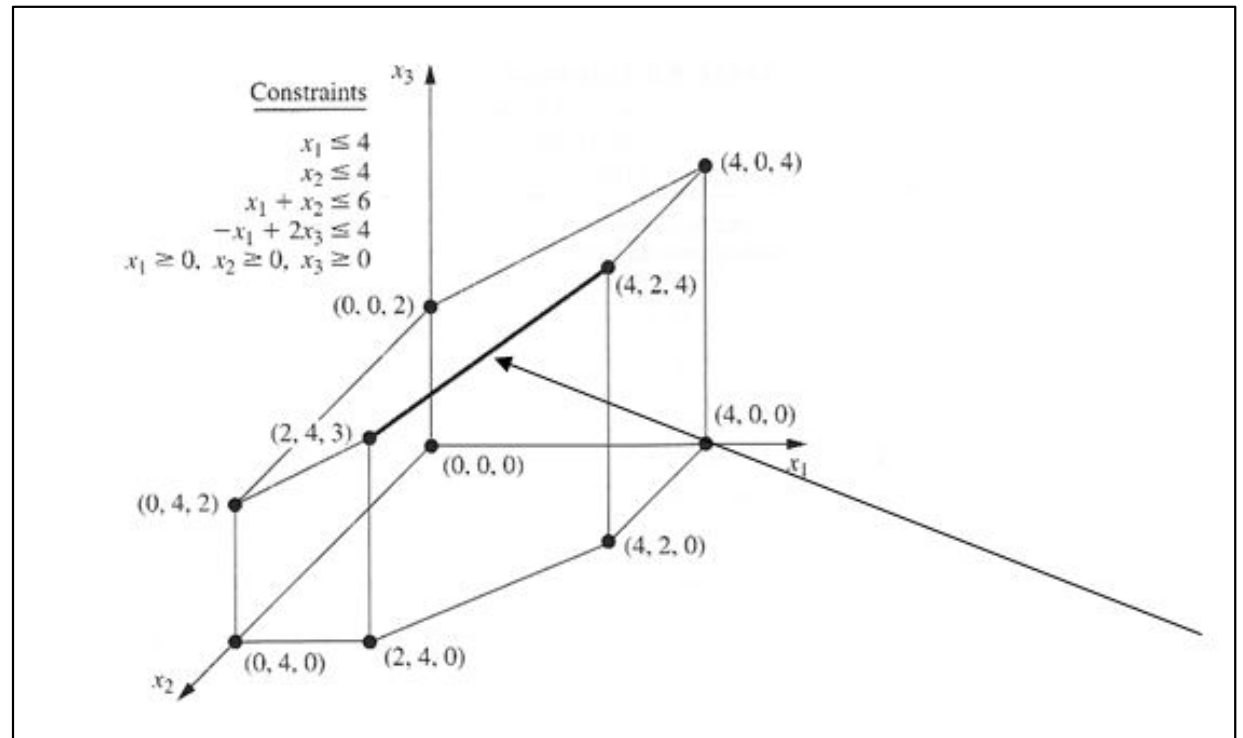
# Two variable simplex search

- Corner-Point Feasible (CPF)
- Constraints and CPFs
- Adjacent CPFs solutions



# Three variables

- Algebraically, we know that each of the CPFs for  $n$  variables, there are  $n$  constraints that create this CPF.
- Moving to an adjacent CPF requires moving across one of the constraints.



# Simplex Method

- Simplex Method and Tabular forms
- Further readings (Hiller, 123-132)



# Computer aided solution

- Excel solver
- CPLEX
- Java libraries
- C++ libraries

# Multi-Objective Optimization

- Goal Programming
- Scoring Models
- Pareto Front Sorting

# Goal Programming

- Goal Programming is an iterative LP model that allows for solving multi-objectives (goals).
- In each iteration, one goal is met.
- The process is repeated until we find the optimal solution that meets all objectives.

# Goal Programming

- Objectives are categorized into primary and secondary goals (priorities). The goals are ranked from  $P_1$  to  $P_n$ .
- Goals are modeled as constraints with deviation variables ( $d$ ).
- Deviation variables allow for not meeting the target value
- The objective is then to minimize the deviation variables.
- Each time a goal is met, its corresponding deviation variable is constrained in the next iteration.

# Illustrative example

It is required to invest up to \$80,000 in stock shares from both UK Oil and Hub Properties to maximize the profit. The maximum accepted risk is 700 (Risk Index). The minimum accepted profit is \$9000. Assume that risk is minimization is far more important to the company than meeting the profit.

<b>Stock</b>	<b>Price per share</b>	<b>Estimate Annual Return per share</b>	<b>Risk Index per share</b>
UK Oil	\$25	\$3	0.5
Hub Properties	\$50	\$5	0.25

# Illustrative Example

- Primary Goal ( $P_1$ ): find stock shares that has 700 Risk index or less
- Secondary Goal ( $P_2$ ): find stock share that provide a minimum of \$9000
- Budget Constraint: Both shares cannot cost more than \$80,000

# Illustrative Example

- Variables:
  - U = Number of UK Oil Stocks
  - H = Number of Hub Properties Stocks

- Cost Constraint:

$$25U + 50H \leq 80,000$$

# Illustrative Example

- Primary Goal Modeling
  - Remember, we want that the total risk index to be less than or equal 700 units.

$$0.5U + 0.25H - d_1^+ + d_1^- = 700$$

*Where:  $d_1^+$  and  $d_1^-$  are the deviation variables;  
how less or more can we deviate from this goal.*

Note: the more  $d^+$  increases, the more risk.



# Illustrative Example

- Secondary Goal Modeling
  - We want that the total profit to be at least equal 9000.

$$3U + 5H - d_2^+ + d_2^- = 9000$$

Note: the more  $d^+$  increases, the more profit

# Illustrative Example

- Solving; Primary Goal first

Objective:  $Min d_1^+$

*Funds Constraints:*  $25U + 50H \leq 80,000$

*P<sub>1</sub> Goal:*  $0.5U + 0.25H - d_1^+ + d_1^- = 700$

*P<sub>2</sub> Goal:*  $3U + 5H - d_2^+ + d_2^- = 9000$

$U, H, d_1^+, d_1^-, d_2^+, d_2^- \geq 0$

# Illustrative Example

- Solving; Primary Goal first

Objective:  $Min d_1^+$

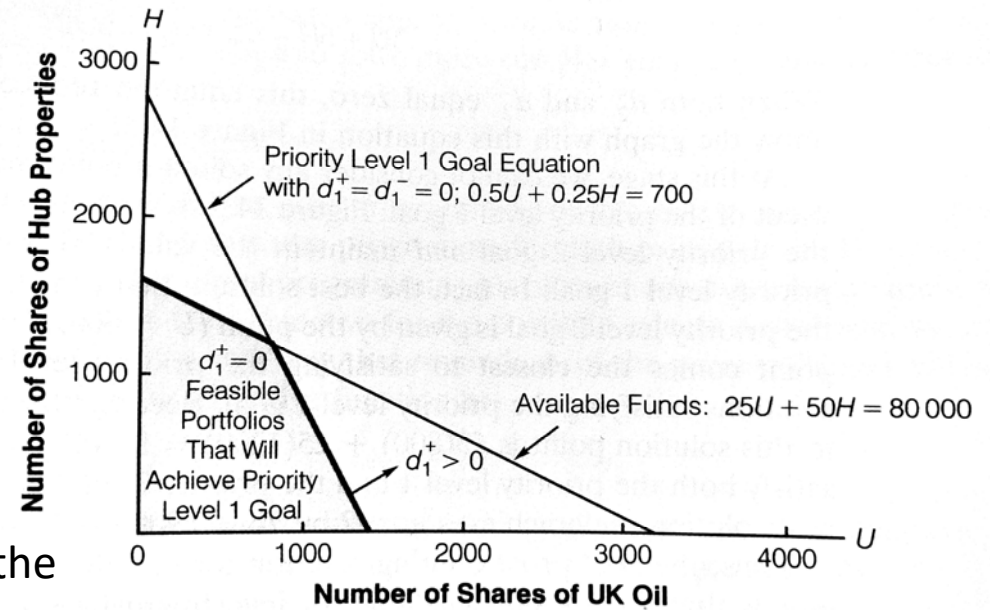
Funds Constraints:  $25U + 50H \leq 80,000$

$P_1$  Goal:  $0.5U + 0.25H - d_1^+ + d_1^- = 700$

$P_2$  Goal:  $3U + 5H - d_2^+ + d_2^- = 9000$

$U, H, d_1^+, d_1^-, d_2^+, d_2^- \geq 0$

- When  $d_1^+, d_1^- = 0$ , The goal is met within the feasible region. Thus, it is optimal.



# Illustrative Example

- Solving; Secondary Goal

Objective:  $Min d_2^-$

*Funds Constraints:*  $25U + 50H \leq 80,000$

*P<sub>1</sub> Goal:*  $0.5U + 0.25H - d_1^+ + d_1^- = 700$

*P<sub>2</sub> Goal:*  $3U + 5H - d_2^+ + d_2^- = 9000$

*Maintain P<sub>1</sub> goal:*  $d_1^+ = 0$

$U, H, d_1^+, d_1^-, d_2^+, d_2^- \geq 0$

# Illustrative Example

- Solving; Secondary Goal

Objective:  $\text{Min } d_2^-$

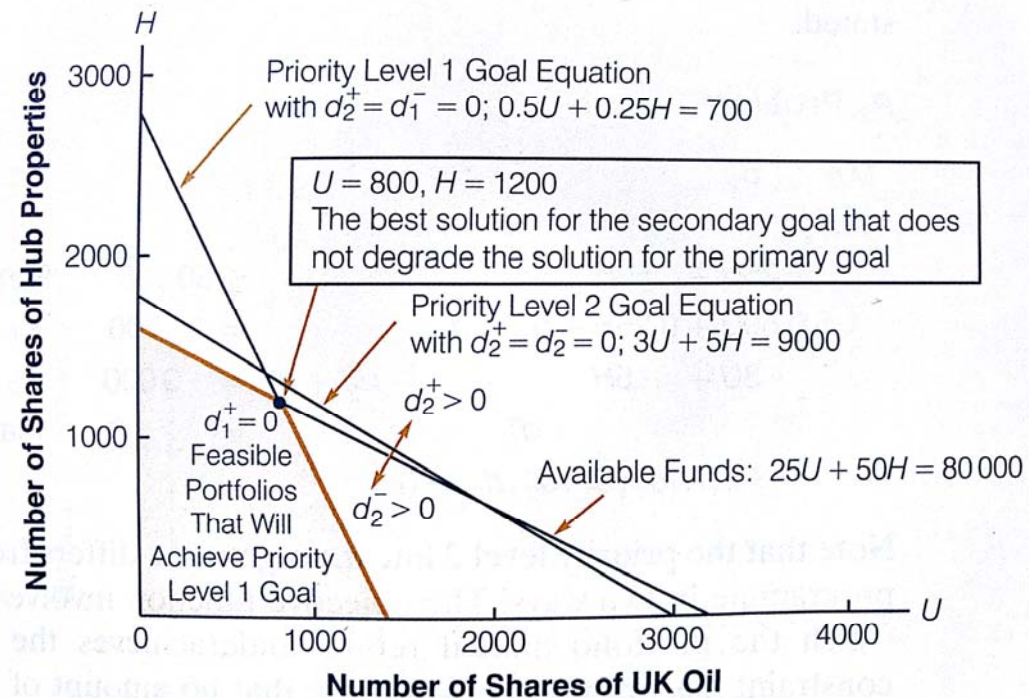
Funds Constraints:  $25U + 50H \leq 80,000$

$P_1$  Goal:  $0.5U + 0.25H - d_1^+ + d_1^- = 700$

$P_2$  Goal:  $3U + 5H - d_2^+ + d_2^- = 9000$

Maintain  $P_1$  goal:  $d_1^+ = 0$

$U, H, d_1^+, d_1^-, d_2^+, d_2^- \geq 0$



# Illustrative Example

- Model

Objective:

$$\text{Min } P_1(d_1^+) + P_2(d_2^-)$$

*s.t.*:

$$25U + 50H \leq 80,000$$

$$0.5U + 0.25H - d_1^+ + d_1^- = 700$$

$$3U + 5H - d_2^+ + d_2^- = 9000$$

$$U, H, d_1^+, d_1^-, d_2^+, d_2^- \geq 0$$

# Notes

- Multiple targets can exist in the same goal
- Steps should be repeated till all goals are met
- Review Anderson et al. (2014) pages 602-607

# Scoring Model

- Scoring models enable for evaluating every alternative (solution) across the number of objectives
- Each objective is weighted depending on its priorities



# Scoring Model

$$S_j = \sum_i w_i * r_{ij}$$

$w_i$  = the weight of each criteria/objective  $i$

$r_{ij}$  = rating of each solution  $j$  for each objective  $i$

$S_j$  = *Score for the solution  $j$*

# Scoring Model Drawbacks

- Subjective weights
  - Analytic Hierarchy Process
- Measuring unit of each objective function

# Pareto Front Sorting

- Pareto Optimal solution is a solution that no other can outperform it
- Pareto optimality concept is used in many fields; economics, engineering, etc.
- Mathematically, what is a Pareto optimal solution?

# Pareto Optimal Solution

$$P = \begin{cases} \min f(x_n) = (f_1(x_n), f_2(x_n), \dots, f_d(x_n)) \\ x_n \in Q, \quad \forall n \in S \end{cases}$$

Where P is the problem domain, d is the number of objective functions, S is the number of solutions, Q is the constraints.

*$x_o$  is Pareto optimal iff*

$$f(x_o) \leq f(x_n) \forall x_n \neq x_o$$

And

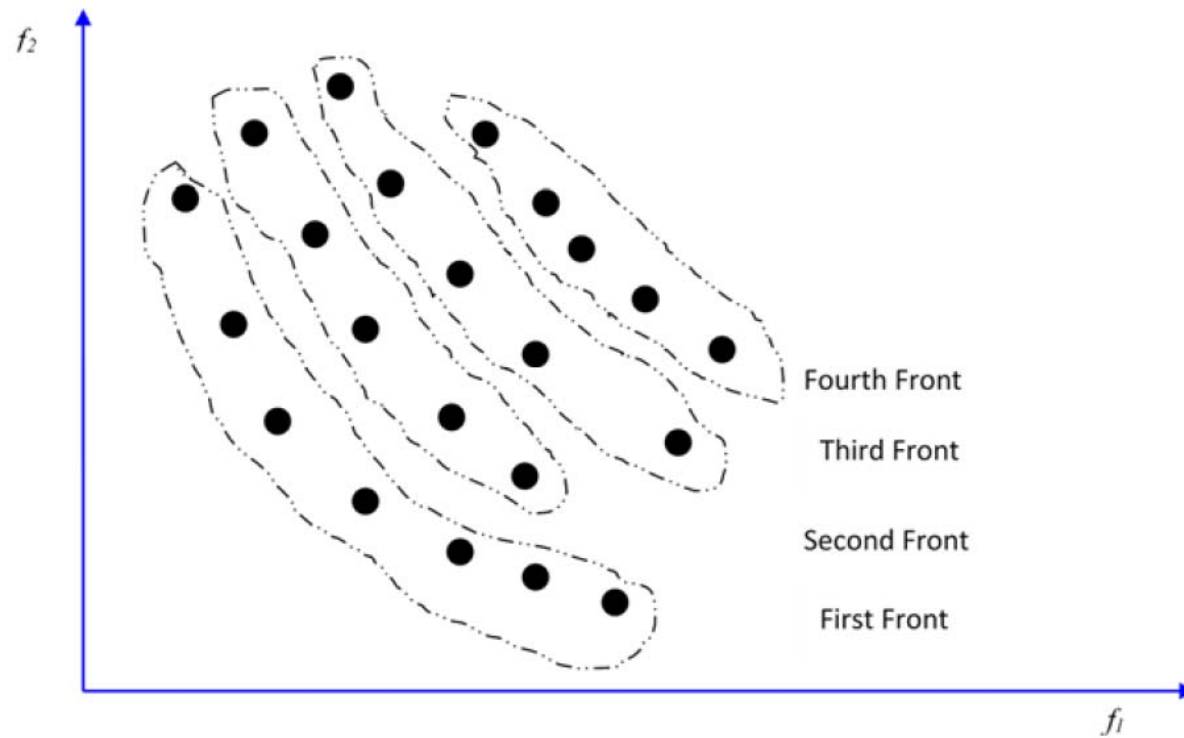
$$\exists x_n \in S \neq x_o, \quad \text{such that } f(x_o) < f(x_n)$$

# Example

- Two minimization functions,  $f_1$  and  $f_2$ .

Solutions	$f_1$	$f_2$
1	5	4
2	2	5
3	2	4
4	6	1
5	3	2
6	4	5
7	3	4
8	3	6

# Pareto Front Sorting



# Pareto Front Sorting

- Can be used for any finite number of functions
  - The same mathematical terms hold true
- Uniform evaluation of the different objectives
- Proposal of multiple/set optimal solutions
- Measuring unit of the evaluation functions