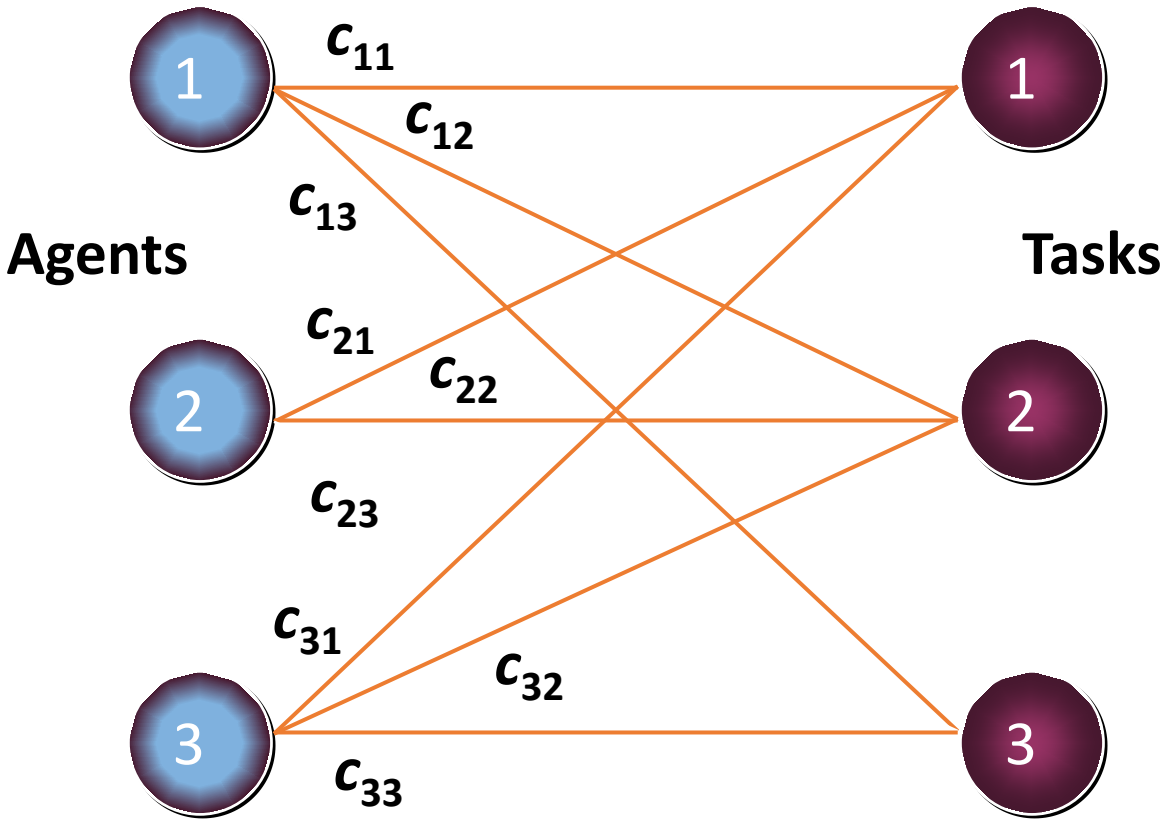
Assignment Problem

Assignment Problem

- An **assignment problem** seeks to minimize the total cost assignment of m workers to m jobs, given that the cost of worker i performing job j is c_{ij}.
- It assumes all workers are assigned and each job is performed.
- An assignment problem is a special case of a transportation problem in which all supplies and all demands are equal to 1; hence assignment problems may be solved as linear programs.
- The network representation of an assignment problem with three workers and three jobs is shown on the next slide.

Assignment Problem

Network Representation



Assignment Problem

- **LP Formulation**

$$\text{Min} \quad \sum_i \sum_j c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_j x_{ij} = 1 \qquad \text{for each agent } i$$

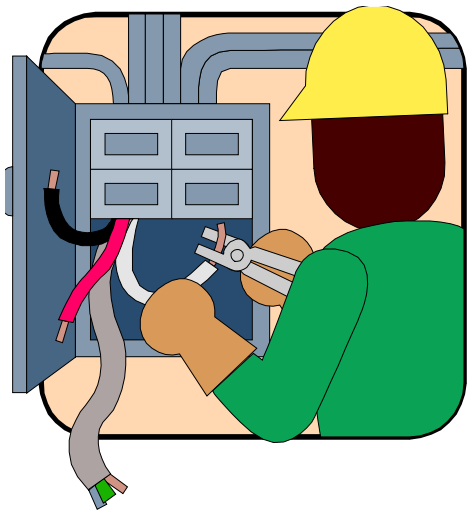
$$\sum_i x_{ij} = 1 \qquad \text{for each task } j$$

$$x_{ij} = 0 \text{ or } 1 \qquad \text{for all } i \text{ and } j$$

Illustration: Who Does What?

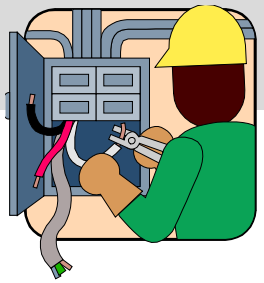
An electrical contractor pays his subcontractors a fixed fee plus mileage for work performed. On a given day the contractor is faced with three electrical jobs associated with various projects. Given below are the distances between the subcontractors and the projects.

	Projects		
Subcontractor	A	B	C
Westside	50	36	16
Federated	28	30	18
Goliath	35	32	20
Universal	25	25	14



How should the contractors be assigned to minimize total mileage costs?

Illustration: Who Does What?



Network Representation

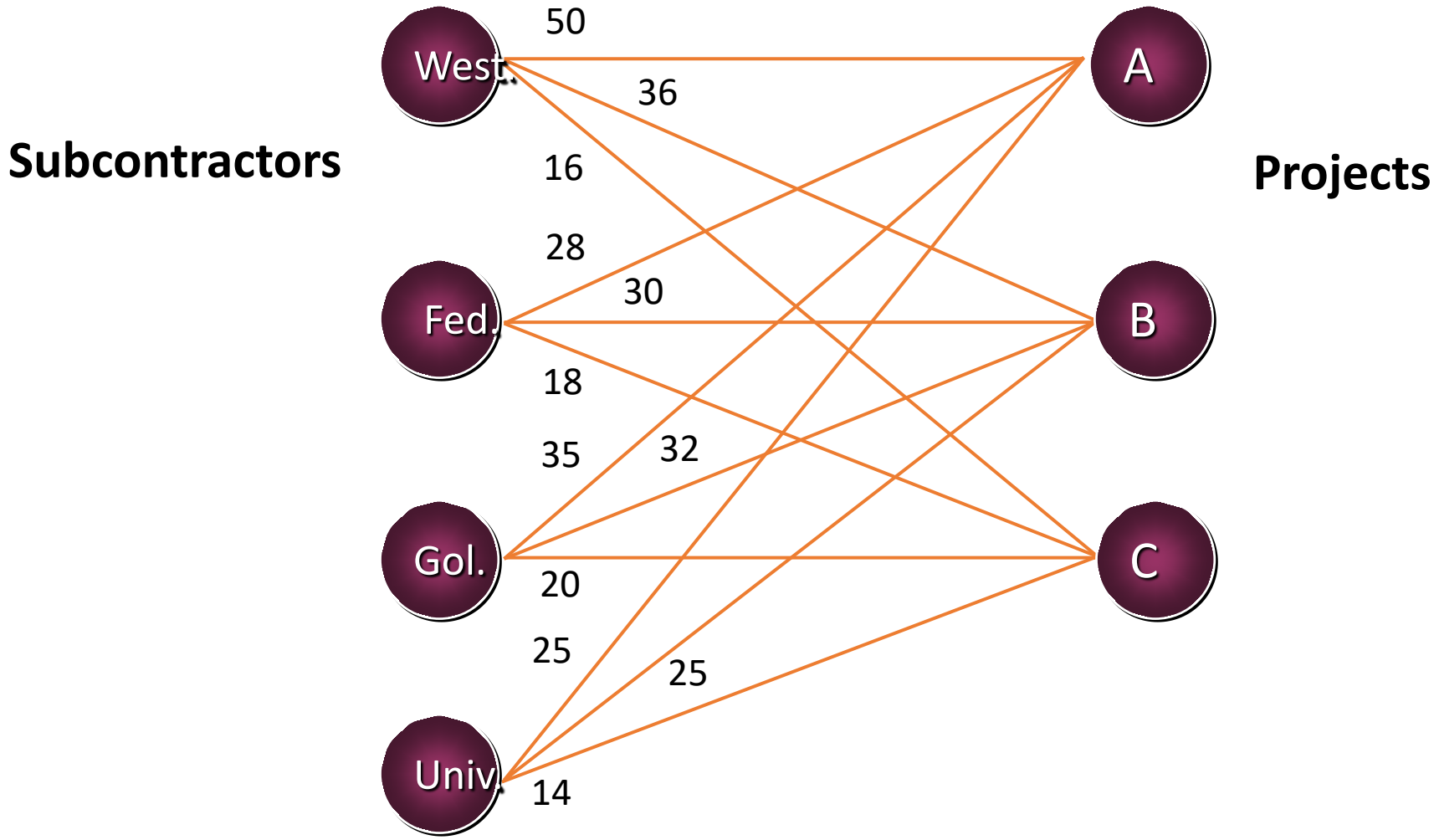
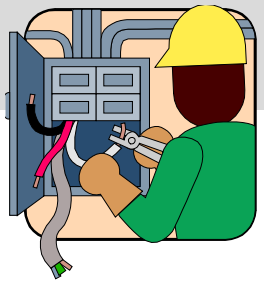


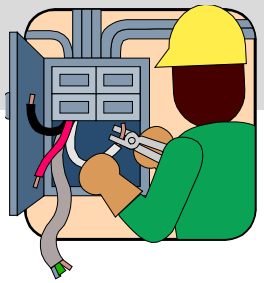
Illustration: Who Does What?



Linear Programming Formulation

$$\begin{aligned}
\text{Min} \quad & 50x_{11}+36x_{12}+16x_{13}+28x_{21}+30x_{22}+18x_{23} \\
& +35x_{31}+32x_{32}+20x_{33}+25x_{41}+25x_{42}+14x_{43} \\
\text{s.t.} \quad & x_{11}+x_{12}+x_{13} \le 1 \\
& x_{21}+x_{22}+x_{23} \le 1 \\
& x_{31}+x_{32}+x_{33} \le 1 \\
& x_{41}+x_{42}+x_{43} \le 1 \\
& x_{11}+x_{21}+x_{31}+x_{41} = 1 \\
& x_{12}+x_{22}+x_{32}+x_{42} = 1 \\
& x_{13}+x_{23}+x_{33}+x_{43} = 1 \\
& x_{ij} = 0 \text{ or } 1 \quad \text{for all } i \text{ and } j
\end{aligned}$$

Illustration: Who Does What?



- **The optimal assignment is:**

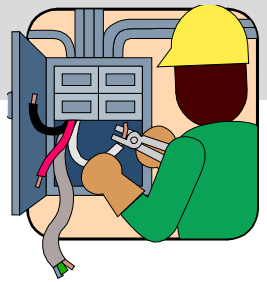
Subcontractor	Project	Distance
Westside	C	16
Federated	A	28
Goliath	(unassigned)	
Universal	B	25

Total Distance = 69 miles

Assignment Problems

- Vogal's Approximation Method

Example 1: Who Does What?



Vogal's Approximation Method

	A	B	C	Dummy
Westside	50	36	16	0
Federated	28	30	18	0
Goliath	35	32	20	0
Universal	25	25	14	0

END of Network Models